Mathematical Portrait
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This document offers a mathematical portrait of Enzo Mitidieri: a concise account of his academic trajectory, the central mathematical problems he has addressed, his principal results and methods, and the broader impact of his work on the theory of Partial Differential Equations and functional inequalities.

1. Biographical and Academic Overview

Enzo Mitidieri is a Full Professor of Mathematical Analysis at the University of Udine, with long-standing ties to the doctoral programme in Mathematics, Earth Sciences, and Fluid Dynamics at the University of Trieste. He belongs to the generation of Italian mathematicians who, from the 1980s onward, placed the Italian school of nonlinear analysis in active dialogue with the Russian and Brazilian communities.

Key academic metrics (ResearchGate, 2025–2026):

Publications	193+
Citations	8,018
***h*-index**	46
Doctoral students	9 (with 15 academic descendants)

These figures position Mitidieri among the foremost Italian analysts in the field of nonlinear PDEs.

2. Mathematical Research Areas

Mitidieri's research is organised around three interconnected themes:

(i) **Existence and nonexistence of solutions to nonlinear elliptic, parabolic, and hyperbolic problems.**

(ii) **Hardy–Rellich functional inequalities and their sharp constants.**

(iii) **Liouville-type theorems and a-priori estimates via nonlinear capacity.**

The unifying thread is the interplay between *criticality* (critical exponents, optimal constants) and *qualitative properties* (positivity, symmetry, blow-up, absence of nontrivial solutions).

3. Central Problems and Main Results

3.1. Nonexistence for Semilinear Elliptic Systems and the Sobolev Hyperbola

A prototype problem studied intensively by Mitidieri is the Lane–Emden system on $\mathbb{R}^N$:

$$-\Delta u = v^p, \qquad -\Delta v = u^q, \qquad u, v > 0 \quad \text{in } \mathbb{R}^N, \tag{1}$$

for exponents $p, q > 0$.

Mitidieri introduced (independently of Van der Vorst) the *Sobolev critical hyperbola*:

$$\frac{1}{p+1} + \frac{1}{q+1} = \frac{N-2}{N}, \tag{2}$$

which separates the existence and nonexistence regimes for (1). Specifically, no positive classical solution exists when (p, q) lies *below or on* the hyperbola (2) (i.e. in the subcritical regime), while solutions may exist above it.

Theorem 3.1 (Mitidieri 1993–1996)**.** *Let $N \geq 3$ and $p, q > 0$. If*

$$\frac{1}{p+1} + \frac{1}{q+1} > \frac{N-2}{N},$$

then system (1) *has no positive radial $C^2(\mathbb{R}^N)$ solution.*

3.2. The Mitidieri–Pohozaev Nonlinear Capacity Method

In a landmark monograph published in the *Proceedings of the Steklov Institute of Mathematics* (2001, 362 pages), Mitidieri and S. I. Pohozaev developed a far-reaching method — now known as the **Mitidieri–Pohozaev nonlinear capacity method** — for proving a-priori estimates and absence-of-solution results for a vast class of nonlinear PDEs and inequalities.

The method proceeds as follows. Consider the model inequality

$$-\Delta_p u \geq |u|^{q-1} u \quad \text{in } \mathbb{R}^N, \tag{3}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ is the p-Laplacian operator, $p > 1$, $q > p-1$, $N > p$.

Key idea. One defines the *nonlinear p-capacity* of a set by testing against a carefully chosen family $\{\varphi_R\}$ of non-negative smooth cut-off functions concentrated on the ball B_R and satisfying

$$\varphi_R(x) = \psi\left(\frac{x}{R}\right), \quad \psi \in C_c^\infty(\mathbb{R}^N), \quad 0 \leq \psi \leq 1.$$

Multiplying (3) by φ_R^m (with m chosen to absorb gradient terms) and integrating by parts yields the fundamental *capacity estimate*:

$$\int_{B_R} |u|^q \varphi_R^m \, dx \leq C \, R^{N - \frac{N(q-p+1)}{q} \cdot \frac{p}{p-1}},$$

which, under the criticality condition

$$q < q^* := \frac{N(p-1)}{N-p}, \qquad (N > p),$$

forces the right-hand side to vanish as $R \to \infty$, thus proving $u \equiv 0$.

Theorem 3.2 (Mitidieri–Pohozaev, 2001)**.** *Let $N > p > 1$. If $p - 1 < q \leq \dfrac{N(p-1)}{N-p}$, then the inequality* (3) *admits no positive weak solution in $\mathbb{R}^N$. The exponent $q^* = \dfrac{N(p-1)}{N-p}$ is sharp.*

The method extends to higher-order operators, systems, parabolic and hyperbolic settings, and nonlocal (integro-differential) equations.

3.3. Hardy–Rellich Inequalities with Sharp Constants

Together with F. Gazzola, H.-Ch. Grunau, and other collaborators, Mitidieri has made deep contributions to the theory of **Hardy–Rellich inequalities**, establishing optimal constants and refined forms with remainder terms.

The classical Hardy inequality in $W_0^{1,p}(\Omega)$, $\Omega \subset \mathbb{R}^N$, reads:

$$\int_\Omega |\nabla u|^p \, dx \;\geq\; \left(\frac{N-p}{p}\right)^p \int_\Omega \frac{|u|^p}{|x|^p} \, dx, \qquad u \in W_0^{1,p}(\Omega), \tag{4}$$

with the constant $\left(\frac{N-p}{p}\right)^p$ being optimal but *not attained*.

In the Gazzola–Grunau–Mitidieri framework, this inequality is refined by adding L^p-remainder terms:

$$\int_\Omega |\nabla u|^p \, dx \;\geq\; \left(\frac{N-p}{p}\right)^p \int_\Omega \frac{|u|^p}{|x|^p} \, dx \;+\; C_{N,p} \int_\Omega |u|^p \, dx,$$

where $C_{N,p} > 0$ is explicit and sharp. For higher-order Sobolev spaces $W_0^{k,p}$, additional remainder terms with lower-order singular weights arise.

The corresponding **Rellich inequality** (second-order analogue) states:

$$\int_{\mathbb{R}^N} |\Delta u|^2 \, dx \;\geq\; \frac{N^2(N-4)^2}{16} \int_{\mathbb{R}^N} \frac{u^2}{|x|^4} \, dx, \qquad u \in C_c^\infty(\mathbb{R}^N \setminus \{0\}), \tag{5}$$

with the constant $\frac{N^2(N-4)^2}{16}$ being sharp. Mitidieri and collaborators have extended (5) to Riemannian manifolds admitting homothetic transformations, proving higher-order polyharmonic Rellich identities via a *Noetherian approach* based on conformal Killing vector fields.

3.4. Mitidieri–Rellich–Pohozaev Identities and Nonexistence on Bounded Domains

The foundational tool introduced in [1] is an integral identity holding for *arbitrary* pairs of smooth functions (u, v), not merely for solutions of a PDE. For $u, v \in C^2(\overline{\Omega})$

and any vector field $h \in C^1(\overline{\Omega}; \mathbb{R}^N)$, writing Du, Dv for the gradients and $(Du, Dv) = \sum_i \partial_{x_i} u \, \partial_{x_i} v$, the identity reads (Corollary 2.1 of [1]):

$$\int_\Omega \{\Delta u \, (h, Dv) + \Delta v \, (h, Du)\} \, dx = \int_{\partial\Omega} \left\{ \frac{\partial u}{\partial n}(h, Dv) + \frac{\partial v}{\partial n}(h, Du) - (Du, Dv)(h, n) \right\} d\sigma$$
$$+ \int_\Omega \operatorname{div} h \, (Du, Dv) \, dx - \sum_{i,j=1}^N \int_\Omega \left\{ \frac{\partial h_j}{\partial x_i} \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_j} + \frac{\partial h_j}{\partial x_i} \frac{\partial u}{\partial x_j} \frac{\partial v}{\partial x_i} \right\} dx \tag{6}$$

Choosing $h(x) = x$ (so $\operatorname{div} h = N$, $\partial h_j / \partial x_i = \delta_{ij}$) yields the key special case (equation (2.5) of [1]):

$$\int_\Omega \{\Delta u \, (x, Dv) + \Delta v \, (x, Du)\} \, dx = \int_{\partial\Omega} \left\{ \frac{\partial u}{\partial n}(x, Dv) + \frac{\partial v}{\partial n}(x, Du) - (Du, Dv)(x, n) \right\} d\sigma$$
$$+ (N-2) \int_\Omega (Du, Dv) \, dx. \tag{7}$$

The classical Rellich identity is recovered by setting $u = v$ in (7).

Applying (7) to solutions of the Hamiltonian system

$$-\Delta u = \frac{\partial G}{\partial v}(u, v), \quad -\Delta v = \frac{\partial G}{\partial u}(u, v) \quad \text{in } \Omega, \qquad u = v = 0 \text{ on } \partial\Omega, \tag{8}$$

with $G(0, 0) = 0$, and using the boundary conditions, yields the Pohozaev-type identity (equation (3.4) of [1]):

$$N \int_\Omega G(u, v) \, dx = (N-2) \int_\Omega (Du, Dv) \, dx + \int_{\partial\Omega} \frac{\partial u}{\partial n} \frac{\partial v}{\partial n}(x, n) \, d\sigma. \tag{9}$$

For a starshaped domain, the boundary term is strictly negative (since $\partial u/\partial n$, $\partial v/\partial n < 0$ on $\partial\Omega$ by the strong maximum principle, and $(x, n) > 0$ by starshapedness). Combined with a convexity condition on G (Theorem 3.1 of [1]), this yields a contradiction, proving nonexistence for system (8).

3.5. Liouville Theorems via Kato's Inequality

In a series of papers with L. D'Ambrosio, Mitidieri proved sharp Liouville theorems for quasilinear degenerate elliptic inequalities using *Kato's inequality for quasi-linear operators*. For the coercive equation

$$-\Delta_p u + |u|^{q-1} u = h \quad \text{in } \mathbb{R}^N, \qquad q > p - 1 > 0, \; N > p,$$

they established *a-priori bounds universal* with respect to the data h, showing that without loss of generality one may assume $u \geq 0$:

Theorem 3.3 (D'Ambrosio–Mitidieri, 2010)**.** *Let $N > p > 1$ and $q > p - 1$. Any weak solution $u \in W^{1,p}_{\mathrm{loc}}(\mathbb{R}^N)$ of $-\Delta_p u \geq |u|^{q-1} u$ in $\mathbb{R}^N$ satisfies $u \equiv 0$ a.e., provided* $p - 1 < q \leq \dfrac{N(p-1)}{N-p}$.

3.6. Liouville Theorems for Fourth-Order Equations

In more recent work (D'Ambrosio–Mitidieri, *Adv. Nonlinear Analysis*, 2022), the authors proved Liouville theorems for the biharmonic inequality

$$\Delta^2 u + f(u) = 0 \quad \text{in } \mathbb{R}^N,$$

where $f(t)t \geq c\,|t|^{q+1}$ for $c > 0$ and $q > 1$. By combining a new version of Hardy–Rellich inequalities with weighted test functions, they obtained nonexistence results *independent of the dimension* N and *independent of the sign* of the solution — a significant strengthening over earlier literature.

4. Principal Collaborators

Mitidieri has built a rich network of international collaborations:

Collaborator	**Institution / Country**
Stanislav I. Pohozaev	Steklov Mathematical Institute, Russia
Djairo G. de Figueiredo	Universidade Estadual de Campinas, Brazil
Lorenzo D'Ambrosio	Università di Bari, Italy
Filippo Gazzola	Politecnico di Milano, Italy
Hans-Ch. Grunau	Universität Magdeburg, Germany
Victor A. Galaktionov	University of Bath, UK
Alberto Farina	Université de Picardie, France
James Serrin	University of Minnesota, USA

5. Selected Major Publications

[1] E. Mitidieri, *A Rellich type identity and applications*, Comm. Partial Differential Equations **18**(1–2), 125–151, 1993.

[2] E. Mitidieri, *Nonexistence of positive solutions of semilinear elliptic systems in* $\mathbb{R}^N$, Differential Integral Equations **9**(3), 465–479, 1996.

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[4] E. Mitidieri, S. I. Pohozaev, *A priori estimates and the absence of solutions of nonlinear partial differential equations and inequalities*, Proc. Steklov Inst. Math. **234**, 1–362, 2001.

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[7] V. A. Galaktionov, E. Mitidieri, S. I. Pohozaev, *Blow-up for Higher-Order Parabolic, Hyperbolic, Dispersion and Schrödinger Equations*, CRC Press, Monographs and Research Notes in Mathematics, 2014.

[8] L. D'Ambrosio, E. Mitidieri, *Entire solutions of certain fourth order elliptic problems and related inequalities*, Adv. Nonlinear Analysis **11**(1), 785–829, 2022.

[9] E. Mitidieri, *A view on Liouville theorems in PDEs*, Analysis and Geometry in Metric Spaces **12**(1), 2024.

6. Impact and Legacy

The **Mitidieri–Pohozaev nonlinear capacity method** has become a standard tool in the analysis of nonlinear PDEs, cited extensively in works on elliptic, parabolic, and hyperbolic equations. The **Sobolev hyperbola** (2) bearing his name (independently introduced) plays a fundamental role in the study of Lane–Emden systems and the Lane–Emden conjecture.

Through 9 doctoral students and 15 academic descendants, and through the organisation of international meetings (including a conference in his honour at his 60th birthday), Mitidieri has built a lasting school in the analysis of nonlinear PDEs bridging Italy, Russia, Brazil, and beyond.

His 2024 paper *"A view on Liouville theorems in PDEs"*, dedicated to Ermanno Lanconelli, demonstrates that at the frontier of his career he continues to survey, synthesize, and extend the field he helped to shape.

This portrait was prepared in March 2026 drawing on ResearchGate, MathSciNet, and published research articles. Citation counts follow ResearchGate (2025–2026).

References

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[3] F. Rellich, *Darstellung der Eigenwerte von* $\Delta u + \lambda u = 0$ *durch ein Randintegral*, Math. Z. **46** (1940), 635–636.

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